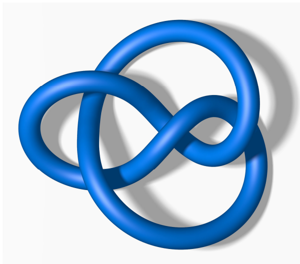
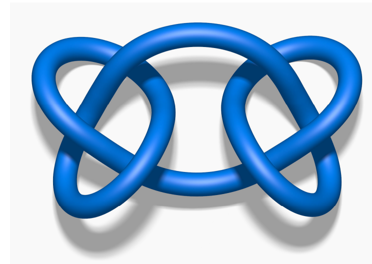


Knot Theory



Michael Polyak

Knots are quite simple objects, so one would expect their theory to be all done and closed by now. However, (and this should not come as a surprise to anybody who once struggled with his badly tied shoelaces), they still resist the power of modern mathematics and conceal as many problems as a century ago.

I plan to start from standard constructions in knot theory and work my way towards more recent developments.

Depending on time and the audience, in the end I may either slide into some new developments in knot theory (e.g., Knot Floer homology) or move on to 3-manifolds. In the latter case, I plan to discuss various constructions of 3-manifolds (handle decompositions and surgery, triangulations and Heegard decompositions) and then carry on with invariants of 3-manifolds.

There are no prerequisites for this course, but a knowledge of linear algebra, groups given by generators and relations, and some basic topology (in particular, fundamental groups and covering spaces) would be definitely helpful.

Tentative syllabus:

Classical knot theory

- Knots, links, and diagrams; isotopy and regular isotopy
- Writhe and linking numbers; Reidemeister moves and colorings
- Knot group and quandle;
- The Alexander polynomial: Fox calculus; Seifert surfaces; Abelian coverings
- Skein relations: Alexander-Conway, Jones, and HOMFLY polynomials
- Other knotted objects: braids, string links, tangles

Modern developments

- Khovanov's categorification of the Jones polynomial
- Vassiliev invariants; chord diagrams; bracelets and claspers
- Gauss diagrams and virtual knots
- * 3-manifolds and their invariants
- * Knot Floer homology

Recommended literature:

1. D. Rolfsen, *Knots and Links*
2. W.B.R. Lickorish, *An introduction to knot theory*